APS Homework 2: Greedy Method

Problem 1: Maximum Subset Product

You are given a list of n integers that can be positive, negative, or 0. Find a subset of the elements that maximizes the product of the elements of the subset. For example, when given the list [0,-2,4,1], there are 2 possible optimal solutions: [4,1] and [4] (either is fine).

***Problem 1a*:** Describe a greedy algorithm for finding a product-maximizing subset given any arbitrary list of integers.

***Problem 1b*:** Prove that the algorithm you provided in *Problem 1a* is correct for any list of integers.

Problem 2: Building Bridges

You are a famous architect, and you have been given the task of building bridges to connect n islands. Specifically, you must build enough bridges such that, given any two islands u and v, there exists some path across the bridges you have built allowing you to go from u to v. In other words, if I start at any island, I should be able to go to any other island using the bridges you have built. A single bridge can connect two islands, so we can denote a single bridge from island u to island v as (u, v).

Of course, building bridges isn't cheap, and you want to try to minimize the overall cost of the project. Let $c(u, v)$ denote the cost of building the bridge (u, v), i.e., a bridge from island u to island v. The overall cost of the project is the sum of the cost of every bridge you built.

Thus, to summarize, there are n islands, I need to be able to travel from any arbitrary starting island to any destination island using the bridges you built, and the sum of the costs of all the bridges you built must be minimized.

***Problem 2a*:** Describe a greedy algorithm for determining which bridges to build such that the islands are connected (i.e., I can travel from any island to any island using the bridges) and such that the total cost of building all the bridges you chose is minimized.

***Problem 2b*:** Prove that the algorithm you provided in *Problem 2a* is correct for any arbitrary n and for any arbitrary set of bridge costs c.